

**YAHYA OULD HAMIDOUNE,
THE MAURITANIAN MATHEMATICIAN**

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Yahya ould Hamidoune passed away in Paris on Friday March 11, 2011 after a brief illness, leaving insufficient time for his friends and colleagues to express their indebtedness to him for his kindness and generosity, both in mathematics and in everyday life. Yahya was a discreet individual, always looking for the essential rather than the superficial, and certainly did not receive the recognition he deserved. May this modest testimony render justice to this singular man.

Yahya ould Hamidoune was born in October 1947 in Atar, Mauritania. His family belong to the highly literate tribe of Oulad Daymân and his father was a famous encyclopædist, writing among other things *the* book on Mauritania, an encyclopædia in 42 volumes, *La vie mauritanienne* (Mauritanian Life).

At fifteen, Yahya went to Cairo to study mathematics up to the graduate level. In 1970, he returned to Nouakchott and began teaching at the Lycée National, the most famous high school in Mauritania; at that time, there was no university in Nouakchott. But Yahya needed challenges, and loved playing games. He became the national champion of Mauritanian draughts. He also attained a high level in chess (as I recall, his ELO was about 2200), tarot, bridge and backgammon. At that time, Yahya was also involved in several revolutionary and anti-neocolonialist movements which shook Mauritanian society, and indeed the entire world, in the 70's. He would pay the price for his involvement, spending several months in jail.

In 1975, Yahya sought a fresh intellectual challenge. He went to France and started his doctoral studies in graph theory with M. Las Vergnas at the University Pierre et Marie Curie (Paris 6). Yahya's first publication, *Sur les atomes d'un graphe orienté*, appeared in the *Comptes Rendus de l'Académie des sciences* in 1977. He soon became an expert on graph connectivity and obtained his PhD (*Quelques problèmes de connexité dans les graphes orientés*) in February 1978. He was admitted to the CNRS (Centre National de la Recherche Scientifique) as a full-time researcher in 1979, and joined C. Berge's group at Paris 6.

Until the middle of the 80's, Yahya worked almost exclusively on connectivity problems. He developed a theory for directed graphs parallel to W. Mader's theory of fragments and atoms in graphs. Among other achievements, he gave in 1978 a proof of the Behzad–Chartrand–Wall conjecture in the case of vertex-transitive graphs. At that time, Yahya was also interested in combinatorial games and matroids, and published a good number of papers on these topics.

After reading H. B. Mann's book *Addition Theorems*, dealing with the theory of sumsets

$$\mathcal{A} + \mathcal{B} = \{a + b, a \in \mathcal{A}, b \in \mathcal{B}\},$$

for \mathcal{A} and \mathcal{B} two subsets of an additive monoid, Yahya realized that certain results of his on graph connectivity generalize, in a disguised form, some classical combinatorial results of additive number theory (at that time, this body of work was not yet called *additive combinatorics*). The *isoperimetric method* was born. Yahya proceeded to investigate, in

a systematic way, the classical theorems of the theory, those due to Cauchy-Davenport, Chowla, Olson, Mann, Shepherdson, Shatrowsky, Vosper, Kneser, Kemperman, He found new proofs, improvements, generalizations, and many applications of these theorems.

Yahya's most famous result is certainly his 1991 proof with J. A. Dias da Silva of a conjecture due to P. Erdős and H. A. Heilbronn on restricted addition of sets modulo a prime p . The paper appears in the *Bulletin of the London Mathematical Society* (1994) under the slightly cryptic title *Cyclic spaces for Grassmann derivatives and additive theory*. The main theorem states that

$$|h^{\wedge} \mathcal{A}| \geq \min(h|\mathcal{A}| - h^2 + 1, p)$$

where $h^{\wedge} \mathcal{A} = \{a_1 + \dots + a_h, a_1, \dots, a_h \in \mathcal{A}, a_i \neq a_j \text{ for all } 1 \leq i \neq j \leq h\}$.

Yahya particularly appreciated challenges, which explains why he loved studying questions posed by others, finding their potential applications of his beloved isoperimetric method.

Here is another, partly more recent, example of Yahya's achievements. If G is an abelian group, the *critical number* of G is the smallest integer k such that any subset \mathcal{S} of G of cardinality at least k satisfies the property that any element of G can be expressed as the sum of the elements of a subset of \mathcal{S} . In a paper coauthored with W. Gao in 1999, Yahya proved the full (that is, for any abelian group) Diderrich conjecture (1975) on the values of these constants. More recently (2008), with A. Lladó and O. Serra — Yahya's principal coauthors — he even answered an analogous but more complicated question of V. Vu dealing with the special case where \mathcal{S} is a set of invertible elements of a cyclic group.

Very recently, Yahya answered in a brilliant way a question of T. Tao dealing with a non-commutative version of Kneser's theorem [1]. Indeed, as I can certify, Yahya was able to answer the question very rapidly after reading it. It is likely that this or similar results already existed in his mind before the question was asked. He simply needed an opportunity to write it down!

In 2001, Yahya was awarded the Chinguitt Prize (from the hands of the President of Mauritania) for his life achievements in science. This is the highest scientific prize in Mauritania, and he was the very first one to receive it. Immediately after obtaining the award, Yahya used it to promote scientific research and teaching in Mauritania. Yahya was very concerned with the development of the sciences in his country. He proposed several improvements in the teaching system, in particular for the organization of high-level teaching within the country. Very recently, Yahya proposed that I accompany him to Mauritania in order to promote the international entrance examinations to the École polytechnique among the best Mauritanian students, *for the love of humanity*, as he loved to say.

But Yahya was not only a mathematician, especially when in Mauritania, where he travelled two or three times a year. There, he was a personality, known to — and loved by — a large part of the population. When his remains arrived in Nouakchott in the middle of the night, about five thousand people were waiting for him at the airport. In fact, Yahya was well-known in Mauritania as a militant for democracy and ecology. He was a deeply honest citizen who did not fear to fight against corruption, a fight which occasionally provoked death threats. One of his preferred combats was protecting the Parc National du Banc d'Arguin — a huge Mauritanian bird sanctuary protected by UNESCO — notably in 2005 against a petroleum company. In fact, Yahya was the main character in a documentary, *Between the oil and the deep blue sea* [2], which shed light on this fight.

I will treasure memories of Yahya ould Hamidoune. As a mathematician, that is evident! But there are not many who can tell you that *mathematics is done to enhance friendship* without appearing ridiculous. Yahya was an independent spirit with high human qualities and high ethical standards. I am certainly not the only one who misses him.

A number of tributes to Yahya have taken place recently, including a one-day event at University Pierre et Marie Curie on March 29 [3]. Several websites contain interesting information about him, as well as photographs [4, 5]. During a UNESCO conference on mathematics in Africa in mid-April, 2011, C. Villani evoked Yahya's character as an example for the African mathematical community. A *Yahya ould Hamidoune Prize* will be created to reward brilliant young Mauritanian students. And a special issue of the *European Journal of Combinatorics* will be dedicated to Yahya's memory. Last, but not least, an international conference in additive combinatorics will be organized at the Institut Henri Poincaré in Paris from July 9 to July 13, 2012 [6].

The interested reader will find a more complete account of Yahya's life and achievements in [7].

Note : The author is grateful to the Editors of *Combinatorics, Probability and Computing* — a journal much appreciated by Yahya, and where he published twelve papers (the revision of the thirteenth being in progress) — for offering him the opportunity to write this tribute to Yahya ould Hamidoune. He also thanks J. A. Bondy for his help in producing a readable text.

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