LOCAL AND GLOBAL TAMENESS IN KRULL MONOIDS

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Let H be a Krull monoid with finite class group G, and let $u \in H$ be an atom (an irreducible element). Then the local tame degree $\mathsf{t}(H, u)$ is the smallest integer $N \in \mathbb{N}_0$ with the following property: for any multiple a of u (so, for any $a \in uH$) and any factorization $a = v_1 \cdot \ldots \cdot v_n$ of a into atoms, there is a short subproduct which is a multiple of u, say $v_1 \cdot \ldots \cdot v_m$, and a refactorization of this subproduct which contains u, say $v_1 \cdot \ldots \cdot v_m = uu_2 \cdot \ldots \cdot u_\ell$, such that $\max\{\ell, m\} \le N$.

Thus the local tame degree $\mathsf{t}(H, u)$ measures the distance between an arbitrary factorization of a and a factorization of a which contains the atom u. By definition, the atom u is a prime element if and only if $\mathsf{t}(H, u) = 0$. The (global) tame degree $\mathsf{t}(H)$ is the supremum of the local tame degrees over all atoms $u \in H$. Again we get that the monoid H is factorial if and only if $\mathsf{t}(H) = 0$. Moreover, the finiteness of the class group easily implies that the finiteness of the tame degree.

We discuss upper and lower bounds for $\mathsf{t}(H)$, and the relationship between $\mathsf{t}(H)$ and the tame degree $\mathsf{t}\big(\mathcal{B}(G)\big)$, where $\mathcal{B}(G)$ is the monoid of zero-sum sequences over the class group G.

This is about joint work with W. Gao and Wolfgang A. Schmid.

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