Journées palaisiennes de combinatoire additive

Palaiseau Days on Additive Combinatorics

Palaiseau, 29 et 30 juin 2017

Palaiseau, June 29 and 30, 2017

Le programme prévisionnel prévoit 14 exposés selon le planning suivant :

There will be 14 talks. Here is the timetable :

Journée du jeudi 29 juin / *Thursday 29 June*

10 :00 – 10 :15		**Accueil** / *Welcome*
10 :15 – 10 :45	O Ramaré	Products of primes in arithmetic progressions
10 :50 – 11 :20	JL Verger Gaugry	Parry Upper functions at real algebraic numbers and Lehmer's Conjecture
11 :20 – 11 :40		**Pause** / *Coffee break*
11 :40 – 12 :10	PY Bienvenu	The polynomial method with application to arithmetic combinatorics in function fields
12 :15 – 12 :35	Q Zhong	The set of minimal distances and the characterization of class groups in Krull monoids
12 :35 – 14 :00		**Pause déjeuner** / *Lunch*
14 :00 – 14 :40	A de Roton	Small sumsets in the circle
14 :45 – 15 :25	I Shkredov	Sum-product phenomenon and sumsets
15 :25 – 15 :45		**Pause** / *Coffee break*
15 :45 – 16 :15	N Hegyvári	Topics in subset sums
16 :20 – 16 ;50	L Rimanic	Szemerédi's theorem in the primes

Journée du vendredi 30 juin / *Friday 30 June*

10 :00 – 10 :15		**Accueil** / *Welcome*
10 :15 – 10 :45	E Balandraud	Addition Theorems via the Combinatorial Nullstellensatz
10 :50 – 11 :20	S Stevens	Point line incidences over finite fields
11 :20 – 11 :40		**Pause** / *Coffee break*
11 :40 – 12 :20	A Zuk	Spectra of automata related to physics
12 :20 – 14 :00		**Pause déjeuner** / *Lunch*
14 :00 – 14 :30	W Schmid	On the Erdős–Ginzburg–Ziv constant for some groups of rank three
14 :35 – 15 :15	Y Fan	Arithmetic properties of the power monoid of $(\mathbb{N}, +)$
15 :15 – 15 :40		**Pause** / *Coffee break*
15 :40 – 16 :20	S Tringali	Periodic properties of directed families with applications to factorization theory

Éric Balandraud : *Addition Theorems via the Combinatorial Nullstellensatz*

In this talk, we will shortly present a direct and reverse way to develop the polynomial method that relies on the Combinatorial Nullstellensatz. The direct (and usual) way states that a multivariate polynomial of small degree cannot vanish on a large cartesian product provided that a specified coefficient is non zero. The reverse way relies on the Coefficient Formula and establishes an expression for this specified coefficient.

This new interpretation of the polynomial method allows to shorten the proofs of the Cauchy-Davenport and the Dias da Silva-Hamidoune Theorem and a new result on the cardinality of sets of subsums. Moreover these proofs do not require any computation and do imply the critical cases of these three problems : arithmetical progressions.

Pierre-Yves Bienvenu : *The polynomial method with application to arithmetic combinatorics in function fields*

We describe a version of the Croot-Lev-Pach polynomial method that had great success last year in arithmetic combinatorics and an application in the function field setting.

Yushuang Fan : *Arithmetic properties of the power monoid of* $(\mathbb{N}, +)$

Let $\mathcal{P}_{\text{fin},0}(\mathbf{N})$ be the monoid of all non-empty, finite subsets of the non-negative integers containing zero with the operation of set addition. We say that a set $A \in \mathcal{P}_{\text{fin},0}(\mathbf{N})$ is an atom if $A = X + Y$ for some $X, Y \subseteq \mathbf{N}$ only if $X = \{0\}$ or $Y = \{0\}$. Given $X \in \mathcal{P}_{\text{fin},0}(\mathbf{N})$, we denote by $\mathsf{L}(X)$ the set of all $k \in \mathbf{N}^+$ such that there exist atoms $A_1, \ldots, A_k$ such that $X = A_1 + \cdots + A_k$, in which case we refer to the unordered k-tuple $(A_1, \ldots, A_k)$ as a factorization of X and to k a (factorization) length of X. We prove that, for every integer $k \geq 2$, the set of lengths of the (discrete) interval $[0, k]$ is the interval $[2, k]$, and there exists $X \in \mathcal{P}_{\text{fin},0}(\mathbf{N})$ that has only two factorizations, one of length 2 and the other of length k.

Norbert Hegyvári : *Topics in subset sums*

A set $A \subseteq \mathbb{N}$ is said to be *complete* if there exists a threshold number n_0 such that every natural number greater than n_0 is the sum of distinct terms taken from A. This concept was introduced by Erdős in the 60's and there are many results from K.F. Roth, Szekeres, Cassels, Davenport etc. We discuss some old and new results extendig this notion to higher dimension.

Olivier Ramaré : *Products of primes in arithmetic progressions : a footnote in parity breaking*

We prove that, if x and $q \leq x^{1/16}$ are two parameters, then for any invertible residue class a modulo q there exists a product of exactly three primes, each one below $x^{1/3}$, that is congruent to a modulo q.

Luka Rimanic : *Szemerédi's theorem in the primes* Green and Tao famously proved in 2005 that any subset of the primes of fixed positive density contains arbitrarily long arithmetic progressions. Green had previously shown that in fact subsets of the primes of relative density tending to zero contain a 3-term term progression. This was followed by work of Helfgott and de Roton, and Naslund, who improved the bounds on the relative density in the case of 3-term progressions, and Henriot, who extended this result to systems of complexity one. The aim of this talk is to present an analogous result for longer progressions. This is joint work with Julia Wolf.

Anne de Roton : *Small sumsets in the circle*

Given two subsets A and B of the circle $\mathbb{T} = \mathbb{R}/\mathbb{Z}$, Raikov proved that the measure $\mu(A + B)$ of the sumset $A + B$ satisfies $\mu(A + B) \geq \min(1, \mu(A) + \mu(B)$. In this talk we shall describe the sets A and B for which the sumset $A + B$ is close to its smallest possible size $\mu(A) + \mu(B)$. The proof relies on a transfer of Serra-Zémor and Grynkiewicz's results in $\mathbb{Z}/p\mathbb{Z}$ to $\mathbb{T}$. We shall also give some applications of our result. This is a joint work with Pablo Candela.

Ilya Shkredov : *Sum-product phenomenon and sumsets*

We will give a survey on the sum-product phenomenon both in the prime fields and in the real setting and how it is connected with additive problems as representations of multiplicatively closed sets as a sum of two sets, multiplicative growth of sumsets and other questions.

Wolfgang Schmid : *On the Erdős–Ginzburg–Ziv constant for some groups of rank three*

Let $(G, +, 0)$ denote a finite abelian group. For every sufficiently long sequence of elements of G, one will find a subsequence whose sum is 0. Moreover, for possibly still longer sequences one will find a subsequence of length $\exp(G)$ whose terms sum to 0 (at some point some element will have to appear $\exp(G)$ times in the sequence).

Classical questions are, for a given G, what is the smallest constant such that every sequence at least this long will have a zero-sum subsequence. This is the Davenport constant of the group, denote $D(G)$. The analogue constant for subsequences of length $\exp(G)$ is called the Erdős–Ginzburg–Ziv constant, denoted $s(G)$. The bound that guarantees a subsequence of length at most $\exp(G)$ is also of interest and typically denoted by $\eta(G)$.

The problem of determining these and related constants is a direct zero-sum problem. The associated inverse problem is the problem of determining the structure of sequences of length just below the respective constant that do not have the required subsequence. For groups of rank at most two all the above mentioned direct problems are solved completely, and also the inverse problems are very well understood and solved in many cases.

Beyond rank two, the situation is much less clear and many questions remain wide open. In this talk we report on recent joint work with Benjamin Girard on $s(G)$ and $\eta(G)$ for groups of rank three whose smallest cyclic component has order two ; the Davenport constant was known.

Sophie Stevens : *Point line incidences over finite fields*

Given finite point and line sets defined in the real (or complex) plane, we can bound the maximum number of times that these sets can intersect – the Szemerédi-Trotter theorem. This bound has been applied to get bounds on seemingly-unrelated areas, such as sum-product theory. Can we do the same for finite point and lines sets defined over any finite field to say something non-trivial ? Assuming that the point and line sets are not too big, previous literature answers this in the affirmative by using tools from additive combinatorics. I will present a new, stronger bound which circumvents many of these tools, returning to the geometry of the question. I will then give some of the applications of this incidence bound. This is joint work with Frank de Zeeuw.

Salvatore Tringali : *Periodic properties of directed families with applications to factorization theory*

Let $\mathscr{L}$ be a collection of non-empty sets of non-negative integers. Given $k \in \mathbf{N}$, we denote by $\mathscr{U}_k$ the union of all $L \in \mathscr{L}$ with $k \in L$, by ρ the supremum of $\sup L/\inf L^+$ as L ranges over $\mathscr{L}$ (that is, the elasticity of $\mathscr{L}$), and by $\Delta(\mathscr{L})$ the set of all $d \in \mathbf{N}^+$ for which there exist $\ell \in \mathbf{N}$ and $L \in \mathscr{L}$ such that $L \cap [\![\ell, \ell+d]\!] = \{\ell, \ell+d\}$. We take $\delta' := 1$ if $\Delta(\mathscr{L}) = \emptyset$, and $\delta' := \inf \Delta(\mathscr{L})$ otherwise.

We call $\mathscr{L}$ a weakly directed family if for all $L_1, L_2 \in \mathscr{L}$ there is $L \in \mathscr{L}$ with $L_1 + L_2 \subseteq L$. We say that $\mathscr{L}$ has accepted (finite) elasticity if $\rho = \sup L/\inf L^+ < \infty$ for some $L \in \mathscr{L}$; and that $\mathscr{L}$ satisfies the Structure Theorem for Unions when there exist $d \in \mathbf{N}^+$ and $M \in \mathbf{N}$ such that $\mathscr{U}_k \subseteq k + d \cdot \mathbf{Z}$ and $\mathscr{U}_k \cap [\![\inf \mathscr{U}_k + M, \sup \mathscr{U}_k - M]\!]$ is an AP with difference d for all large $k \in \mathbf{N}$.

We show that, if $\mathscr{L}$ is weakly directed and there is $K \in \mathbf{N}$ such that $\sup \mathscr{U}_{k+1} \leq \sup \mathscr{U}_k + K$ and $\inf \mathscr{U}_k - K \leq \inf \mathscr{U}_{k+1}$ for all large k, then $\mathscr{L}$ satisfies the Structure Theorem for Unions with $d = \delta'$.

In addition, we prove that, if $\mathscr{L}$ is a weakly directed family with accepted elasticity, then there are $m \in \mathbf{N}^+$ and finite sets $\mathscr{U}_0', \mathscr{U}_0'', \ldots, \mathscr{U}_{m-1}', \mathscr{U}_{m-1}'' \subseteq \mathbf{N}$ such that, for all large $k \in \mathbf{N}$,

$$\mathscr{U}_k = \left(\inf \mathscr{U}_k + \mathscr{U}'_{k \bmod m}\right) \uplus \mathscr{P}_k \uplus \left(\sup \mathscr{U}_k - \mathscr{U}''_{k \bmod m}\right) \subseteq k + \delta' \cdot \mathbf{Z},$$

where $\mathscr{P}_k$ is an AP with difference δ' (in particular, H satisfies the Structure Theorem for Unions).

The last result applies, in the first place, to the system of sets of lengths of a monoid with accepted elasticity : Most notably, this covers the case (of the multiplicative monoid) of all commutative Krull domains with finite class group (including the ring of integers of a number field), of wide classes of weakly Krull commutative domains (including all orders in algebraic number fields with finite elasticity), and of certain maximal orders in central simple algebras over global fields.

Jean-Louis Verger-Gaugry : *Parry Upper functions at real algebraic numbers and Lehmer's Conjecture*

The minoration of the Mahler measure $\mathrm{M}(\alpha)$ of nonzero algebraic integers α which are not roots of unity is an old problem. The Conjecture of Lehmer states that there exists a universal constant $c > 0$ such that $\mathrm{M}(\alpha) \geq 1 + c$ for such algebraic integers (initially formulated as a problem in 1933 by Lehmer). On the other hand the Rényi-Parry dynamical system given by the β-transformation T_β on the interval $[0, 1]$, with $\beta > 1$ real, provides several functions which depend upon β and are correlated : the Fredholm determinant of the Perron-Frobenius operator, resp. transfer operator (Ruelle, Baladi and Keller), of the beta-transformation, the dynamical (Artin-Mazur) zeta function $\zeta_\beta(z)$ of the β-shift, the Parry Upper function $f_\beta(z)$ at β constructed from the inverse of $\zeta_\beta(z)$. In this study, $\beta > 1$ is the house of a nonzero algebraic integer α which is not a root of unity. We prove that the geometry of some subcollections of zeroes of $f_\beta(z)$ are directly correlated to the minoration problem of the Mahler measure of α and that Lehmer's Conjecture is true for some families of algebraic integers. This study is based on the combinatorics of the moderate lacunarity of $f_\beta(z)$ and the Poincaré asymptotic expansions of the roots of the Parry Upper function.

Qinghai Zhong : *The set of minimal distances and the characterization of class groups in Krull monoids*

Let H be a Krull monoid with finite class group G such that every class contains a prime divisor. Then every non-unit $a \in H$ can be written as a finite product of atoms, say $a = u_1 \cdot \ldots \cdot u_k$. The set $\mathsf{L}(a)$ of all possible factorization lengths k is called the set of lengths of a. There is a constant $M \in \mathbb{N}$ such that all sets of lengths are almost arithmetical multiprogressions with bound M and with difference $d \in \Delta^*(H)$, where $\Delta^*(H)$ denotes the set of minimal distances of H. We study the structure of $\Delta^*(H)$ and characterize the class group for which $\Delta^*(H)$ is an interval.

It is classical that the system $\mathcal{L}(H) = \{\mathsf{L}(a) \mid a \in H\}$ of all sets of lengths depends only on the class group G, and a standing conjecture states that conversely the system $\mathcal{L}(H)$ is characteristic for the class group. We verify the conjecture if the class group is isomorphic to C_n^r with $r, n \in \mathbb{N}$ and $\Delta^*(H)$ is not an interval.

Andrzej Zuk : *Spectra of automata related to physics*

Box-ball systems are discrete analogues of the KdV equation. We prove that their evolution can be described by automata and study their spectral properties.