Machine Copy for Proofreading, Vol. x, y–z, 2010

AUTOMATIC DESIGN OF CIRCULAR SIW RESONATORS BY A HYBRID APPROACH BASED ON POLYNOMIAL FITTING AND SVRMS

D. De Carlo and S. Tringàli

Department of DIMET
University "Mediterranea"
Via Graziella n. 1, Localita Feo di Vito, Reggio Calabria (RC) 89122, Italy

Abstract—We obtain analytical formulas enabling automatic design procedures of circular Substrate Integrated Waveguide (SIW) resonators, assuming a prescribed resonance frequency $f_{\rm res}$. On the one hand such formulas allow us to write flexible codes, on the other hand they yield sharp estimates of the SIW geometrical parameters, which are subsequently passed as inputs to a Support Vector Regression Machine (SVRM), especially trained to make compensation. In particular, the SVRM uses such estimates to compute a further frequency $f'_{\rm res}$, possibly different from $f_{\rm res}$, and the overall process is restarted over and over through a feedback loop, until the relative error $|f_{\rm res} - f'_{\rm res}|/f_{\rm res}$ is reduced under a given threshold. To validate the proposed approach, we finally compare the design outputs provided by its implementation in the MATLAB environment with the results of HFSS simulations.

1. INTRODUCTION

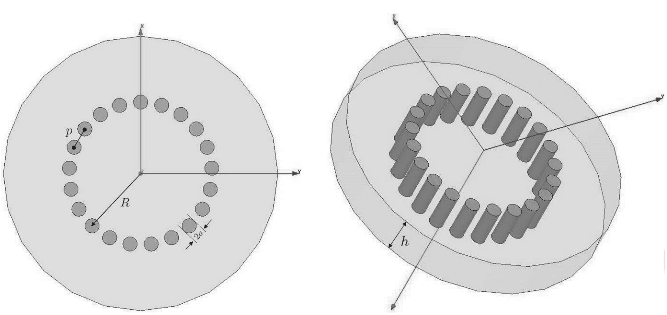
Substrate Integrated Waveguide (SIW) circuits [1, 2] have been attracting much attention in the field of millimetric engineering, as representing an effective alternative to metallic waveguides and resonators due to their simplicity and reduced costs of realization. This is really interesting, since many modern devices like filters are based on resonators [3–5]. In particular, circular SIW resonators are created within a dielectric substrate (usually a soft board) by adding metal layers on the top and the bottom of the structure and "caging" it with a circular fence of plated cylindrical vias of a given radius a, spaced

Corresponding author: S. Tringàli (salvo.tringali@gmail.com).

each other by a fixed distance p, known as *pitch*. If everything works out, the result looks like a dielectrically-filled lossy circular cavity, with reduced height h compared to the standard $2:1$ radius/height ratio, as shown in Figures 1(a) and 1(b).

Anyway technological solutions based on SIWs present a major drawback, in that their design requires a thorough understanding of the functional dependance of the operating frequency f_{res} from the previous geometrical parameters [6–8]. Unfortunately such a dependance is not known explicitly, even if dealing with standard-shaped structures. In other words, while the underlying relationships between resonances and geometrical parameters is well understood as far as standard technological solutions are exploited — in fact they can even be predicted on the basis of closed analytical formulas in some remarkable cases of interest —, nothing similar is available for SIW-based resonators. As a result, their design is usually cumbersome [11].

The canonical case of circular SIW resonators is discussed in some details in [9]. There the authors adopt an approach based on the study of the reciprocal scattering by a finite number of perfectly conducting cylinders buried in a homogeneous space to draw interesting conclusions. In particular, they compute resonant solutions seeking frequencies, which make the smallest singular value of the discretized scattering operator attain its local minima, through the computational method presented in [12], and exploit interpolation to fit such data and plot parametric curves relating resonances to geometrical parameters in a graphical fashion. Hereafter we resume and improve their work, deriving analytical formulas that — combined in a loop of feedback with Support Vector Regression Machines (SVRMs) [14], especially trained to compensate errors stemming from various approximations — enable a completely automatic design of circular SIW resonators.



(a) (b)

Figure 1. (a) 2D transversal and (b) 3D of a circular SIW resonator, showing the geometrical parameters which its design is depending on.

2. THE AUTOMATIC DESIGN PROCEDURE

Let us suppose we want to determine some actual values of the geometrical parameters a, R and p so that the fundamental mode of the circular SIW resonator shown in Figure 1(a) resonates at a given frequency f_{res}. The relative permittivity of the dielectric substrate ε and the height h of the structure are given. We start estimating the radius R by the well-known formulas for an ideal PEC circular cavity. Then we relate the parameter $\chi = Rf_{\mathrm{res}}\sqrt{\varepsilon_r}$ to the pitch p and the radius a of the via holes by means of second-order polynomial fitting formulas as discussed in [9]. This equation describes a conic on the appropriate plane: hence it is verified by infinitely many possible pairs like (a, p). To force a unique solution, we address the common situation in which both the resonance frequency and technological constraints on the plated vias' diameter are given. Under these assumptions the calculations carried out in Section 2.1 provide an analytical procedure to minimize the pitch, according to the physical intuition that the losses through the side walls are consequently minimized [10].

The overall procedure consists of two main steps. First, we exploit basic calculus to establish that p as a function of a attains its global minimum $\hat{p}$ for some $\hat{a} \in [a_m, a_M]$, while writing down analytical formulas that provide quantitative computations of both $\hat{p}$ and $\hat{a}$. In particular, the direct inspection of all the resulting cases gives rise, at most, to eight numerical real values as appropriate. The minimum we are looking for is just among these values. Hence it can be automatically determined by the MATLAB function min(), once that all possible alternatives have been correctly worked out.

Then we pass the so far computed geometrical parameters as inputs to a SVRM, especially trained to make compensation on the approximation errors. In particular, starting from the analytical values

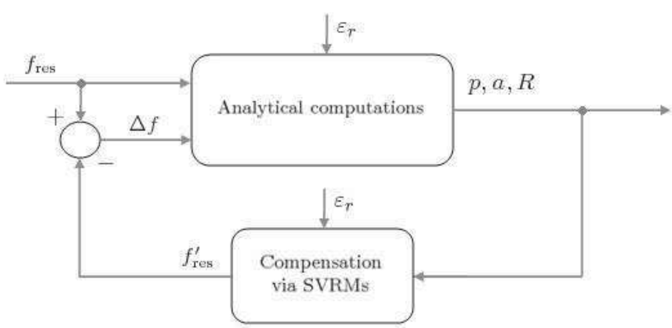


Figure 2. Block representation of the automatic design procedure.

of R, $\hat{p}$ and $\hat{a}$, the SVRM calculates a new resonance frequency $f'_{\rm res}$, possibly different from $f_{\rm res}$, and compares the former and the latter. Then the overall procedure is iterated over and over through a loop of feedback, until the absolute value of the relative error $\Delta f = (f_{\rm res} - f'_{\rm res})/f_{\rm res}$ is reduced under a given threshold, according to the block-diagram representation in Figure 2.

2.1. Analytical Formulas for Computing a and p

Even if the results reported in [9] are restricted to *circular* resonators, the method can be naturally extended to general geometries. In some more details, the cited paper provides formulas involving second-order complete polynomials in the variables p and a in order to relate the pitch p and the radius a of the plated vias of a circular SIW resonator to the operating resonance frequency $f_{\rm res}$ and to the relative permittivity ε_r of the dielectric substrate in terms of the quantity $\chi = R f_{\rm res}\sqrt{\varepsilon_r}$. Such polynomials present the general form

$$c_{2,0}p^2 + c_{1,1}ap + c_{0,2}a^2 + c_{1,0}p + c_{0,1}a + c_{0,0} = \chi \qquad (1)$$

where the coefficients $c_{i,j}$ are tabulated according to the cavity radius R. Though less precise than 2-variate polynomials of higher degree, quadratics are preferred as they require less efforts to be calibrated and are proved to be more robust in fitting procedures. Now, still following [9], once that χ is known, a and p are determined by graphical interpolation, which is a significant limit to any possible automatic implementation of the exploited approach. So overcoming such limitations and having a completely analytical approach may be of great interest. This is actually what we do hereafter.

Without loss of generality, we suppose that a assumes values in the closed interval $[a_m, a_M]$, where $0 < a_m \le a_M$. Once given the design inputs ε_r and $f_{\rm res}$, we compute a reasonable approximation of the cavity radius R, and we use it to evaluate the auxiliary parameter χ. Then we admit both $c_{2,0}$ and $c_{0,2}$ are not null, so that Equation (1) can be effectively regarded as a quadratics either in the variable a or in the variable p. In particular, after reordering it according to the decreasing powers of p, Equation (1) takes the form

$$c_{2,0}p^2 + (c_{1,0} + c_{1,1}a)p + c_{0,2}a^2 + c_{0,1}a + c_{0,0} - \chi = 0 \qquad (2)$$

and can be easily solved for p. Indeed, after computing its discriminant

$$\begin{aligned}\Delta &= (c_{1,0} + c_{1,1}a)^2 - 4c_{2,0}\left(c_{0,2}a^2 + c_{0,1}a + c_{0,0} - \chi\right) \\ &= (c_{1,1} - 4c_{2,0}c_{0,2})\, a^2 + 2\,(c_{1,0}c_{1,1} - 2c_{2,0}c_{0,1})\, a \\ &\quad + c_{1,0}^2 - 4c_{2,0}(c_{0,0} - \chi).\end{aligned} \qquad (3)$$

we use the resolving formula for quadratics to find out that its zeros are given by

$$p_{\pm} = -\frac{c_{1,0} + c_{1,1}a \pm \sqrt{\Delta}}{2c_{2,0}} \tag{4}$$

Then we note that $\Delta = \Delta(a)$ is itself a quadratic polynomial in the variabile a, so a continuous function of a. It is useful to simplify its expression just defining $\alpha_0 = c_{1,1} - 4c_{2,0}c_{0,2}$, $\alpha_1 = c_{1,0}c_{1,1} - 2c_{2,0}c_{0,1}$ and $\alpha_2 = c_{1,0}^2 - 4c_{2,0}(c_{0,0} - \chi)$, so that

$$\Delta = \alpha_0 a^2 + 2\alpha_1 a + \alpha_2. \tag{5}$$

Since p is a length, it has to be real and positive, so $\Delta = \Delta(a)$ must be ≥ 0, for any $a \in [a_m, a_M]$. Now we let $D = \alpha_1^2 - \alpha_0\alpha_1$ and suppose $\alpha_0 \neq 0$. Then some cases have to be distinguished:

(i) $D < 0$ and $\alpha_0 > 0$. Then $\Delta(a) \geq 0$ for any $a \in [a_m, a_M]$, so no further analysis is needed.

(ii) $D < 0$ and $\alpha_0 < 0$. Then $\Delta(a) \geq 0$ is never fulfilled for real values of a, so this case is not significant for applications and should be avoid.

(iii) $D \geq 0$ and $\alpha_0 > 0$. Then $\Delta(a) \geq 0$ if and only if $a \in [a_m, a_M]$ and

$$a \leq \frac{-\alpha_1 - \sqrt{D}}{\alpha_0} \quad \text{or} \quad a \geq \frac{-\alpha_1 + \sqrt{D}}{\alpha_0} \tag{6}$$

(iv) $D \geq 0$ and $\alpha_0 < 0$. Then $\Delta(a) \geq 0$ if and only if $a \in [a_m, a_M]$ and

$$\frac{-\alpha_1 - \sqrt{D}}{\alpha_0} \leq a \leq \frac{-\alpha_1 + \sqrt{D}}{\alpha_0} \tag{7}$$

In all cases, whenever a solution is possibile, it is found that a ranges in a set A, that is either a closed and bounded interval or the union of two such disjoint intervals. Since $\Delta = \Delta(a)$ is a continuous function, this grants the existence of some $\bar{a} \in A$ such that $p = p(a) = \min(p_-(a),\ p_+(a))$ attains its global minimum for $a = \bar{a}$.

So we have to solve the problem of determining the global minimum of the functions $p_{\pm} = p_{\pm}(a)$ under the hypothesis that a belongs to a closed bounded interval or to the union of two such intervals. Clearly it is sufficient to outline the right procedure in the case of one only interval $[a'_m, a'_M]$. We start computing the first derivatives

$$p'_{\pm}(a) = -\frac{1}{2c_{2,0}}\left(c_{1,1} \pm \frac{\Delta'(a)}{2\sqrt{\Delta(a)}}\right) = -\frac{1}{2c_{2,0}}\left(c_{1,1} \pm \frac{\alpha_0 a + \alpha_1}{\sqrt{\Delta(a)}}\right) \tag{8}$$

Then we solve $p'_\pm(a) = 0$ to calculate the critical points of $p_\pm = p_\pm(a)$. This yields, respectively, the two irrational equations

$$c_{1,1}\sqrt{\Delta(a)} = \mp(\alpha_0 a + \alpha_1) \tag{9}$$

which split into two subsystems, according to the sign of the right-hand side brackets, i.e.,

$$p'_+(a) = 0 \quad \text{if and only if} \quad \begin{cases} c_{1,1}^2\Delta(a) = (\alpha_0 a + \alpha_1)^2 \\ \alpha_0 a + \alpha_1 \leq 0 \end{cases} \tag{10}$$

and

$$p'_-(a) = 0 \quad \text{if and only if} \quad \begin{cases} c_{1,1}^2\Delta(a) = (\alpha_0 a + \alpha_1)^2 \\ \alpha_0 a + \alpha_1 > 0 \end{cases} \tag{11}$$

These can be further simplified taking

$$q_0 = \alpha_0 c_{1,1}^2 - \alpha_0^2; \quad q_1 = 2\alpha_1 c_{1,1}^2 - 2\alpha_0\alpha_1; \quad q_2 = \alpha_2 c_{1,1}^2 - \alpha_1^2, \tag{12}$$

so that

$$p'_+(a) = 0 \quad \text{if and only if} \quad \begin{cases} q_0 a^2 + q_1 a + q_2 = 0 \\ \alpha_0 a + \alpha_1 \leq 0 \end{cases} \tag{13}$$

and

$$p'_-(a) = 0 \quad \text{if and only if} \quad \begin{cases} q_0 a^2 + q_1 a + q_2 = 0 \\ \alpha_0 a + \alpha_1 > 0 \end{cases} \tag{14}$$

So finally some cases must be analyzed separately. Each of the previous systems provides not more than two distinct solutions for the variable a. Let A^+ be the set of solutions of system (13), and A^- be the set of solutions of system (14). Then we compute the image sets

$$p_\pm(A^\pm) = \left\{p_\pm(a) : a \in A^\pm\right\}, \tag{15}$$

subsequently comparing the resulting values with the ones attained by $p_+(a)$ and $p_-(a)$ at the end-points of the interval $[a'_m, a'_M]$. The minimum exists, since we have to discriminate between a finite amount of positive real numbers, and it corresponds to the minimum pitch we are searching for.

2.2. Using SVRMs to Compensate Approximation Errors

Soft-computing via Support Vector Machines (SVMs) is an active field of research, and there exists a large literature on the topic. Here we just recall its main issues, referring to the bibliography for any further details. SVMs are learning machines performing pattern recognition tasks. They map m-dimensional input data sets into a higher n-dimensional space, where they can be linearly separated (see [14] and references therein). SVMs can also be employed to solve regression problems, which is the case when they specialize in Support Vector Regression Machines (SVRMs). Indeed let us consider the problem of approximating the set of points

$$\mathcal{D} = \{(x_1, y_1), (x_2, y_2), \ldots, (x_k, y_k)\} \subseteq X \times \mathbb{R} \tag{16}$$

by an affine function $f(\mathbf{x}) = \langle \mathbf{w}, \mathbf{x} \rangle + b$, where $\mathbf{w} \in X$ is a fixed vector of weights, $b \in \mathbb{R}$ and $\langle \cdot\,, \cdot \rangle$ is some inner product on the domain X of $f(\cdot)$, i.e., the space of the input patterns. Such a problem can be solved selecting the optimal regression function minimizing the real functional

$$\Phi(\mathbf{w}, \xi) = \frac{1}{2}\|\mathbf{w}\|^2 + C \sum_{i=1}^{m} \left(\xi_i^- + \xi_i^+\right), \tag{17}$$

where m is the algebraic dimension of X as a vector space over the real field, C is an absolute constant, and ξ_i^- and ξ_i^+ are slack variables representing upper and lower bounds on the outputs of the system [14]. The problem is that $\mathcal{D}$ often fails to be linearly separable, which means the data can be separated only by some nonlinear transformation, mapping $\mathcal{D}$ into a higher dimensional space, where a linear regression happens to be possible. This is accomplished by the choice of a suitable *kernel function* $K(\cdot)$, which represents the core of any SVRM and regulates its effectiveness [15]. The task is to find a functional form which provides the SVRM with the right prediction accuracy.

According to the specific application, the system can "learn" the unknown functional relation between inputs and outputs from experience. Hence data sets are needed for its training on a case-by-case basis. Furthermore, a loss function is necessary. In particular, our approach is based on Vapnik's ε-insensitive loss function

$$L_\varepsilon(\mathbf{x}) = \begin{cases} 0 & \text{if } |f(\mathbf{x}) - y| < \varepsilon \\ f(\mathbf{x}) - y & \text{otherwise} \end{cases}. \tag{18}$$

SVRMs exploiting these ideas have been implemented and tested in the MATLAB environment with the support of the Spider Toolbox [16], following the recipes given in [13].

3. RESULTS

In order to validate our approach, different projects have been considered. Some of them are summarized in Table 1. In all cases the pitch p is assumed to be minimized under the constraint that the radius a of the plated vias ranges in a certain interval. Once given the target frequency f_{res}, our codes return the geometrical parameters of a structure, that is expected to resonate at the desired frequency. Subsequently, this structure is implemented in the framework of HFSS and a second frequency f_{HFSS} is calculated. Finally f_{res} and f_{HFSS} are compared in terms of the absolute percent error $\delta f = |f_{\mathrm{res}} - f_{\mathrm{HFSS}}|/f_{\mathrm{res}}$. The results are reported in Table 2, where a good agreement is shown between full-wave simulations and our predictions. Finally, it is notable that, once the SVRM has been trained (see [13] for details on the training time), the compensation step is almost immediate. This makes the approach especially suitable for iterated applications.

Table 1. SIW resonators obtained from the automatic synthesis procedure imposing the condition of the minimum pitch.

	f_{res}	ε_r	h [mm]	a [mm]	R [mm]	n	p [mm]
Structure 1	24.14	2.2	0.50	0.76	2.40	15	1.60
Structure 2	16.27	9.9	0.38	0.23	2.40	37	0.41
Structure 3	26.39	2.2	0.50	0.21	4.83	75	0.41

Table 2. Absolute percent error on the design frequency made by the automatic synthesis procedure with reference to the cases summarized in Table 1.

	f_{res}	f_{HFSS}	δf
Structure 1	24.14	23.88	1.08%
Structure 2	16.27	16.50	1.41%
Structure 3	26.39	26.03	1.38%

4. CONCLUSION

In this paper, an automatic procedure has been developed for the determination of the resonant frequencies of circular SIW resonators. The proposed approach is based on analytical formulas and Support Vector Regression Machines (SVRMs), which have been trained to make a compensation on the design geometrical parameters through a feedback loop. Results have been compared with HFSS simulations. The maximum observed absolute error is about 1 percent over the design frequency, so proving the effectiveness of the proposed approach.

REFERENCES

1. Deslandes, D. and K. Wu, "Single substrate integration technique of planar circuits and waveguide filters," *IEEE Trans. Microw. Theory Tech.*, Vol. 51, No. 2, 593–596, 2003.
2. Sotoodeh, Z., B. Biglarbegian, F. H. Kashani, and H. Ameri, "A novel bandpass waveguide filter structure on SIW technology," *Progress In Electromagnetics Research Letters*, Vol. 2, 141–148, 2008.
3. Potelon, B., J.-C. Bohorquez, J.-F. Favennec, C. Quendo, E. Rius, and C. Person, "Design of ku-band filter based on substrate integrated circular cavities (SICCs)," *IEEE MTT-S Int. Dig.*, 1237–1240, 2006.
4. Tang, H. J., W. Hong, Z. C. Hao, J. X. Chen, and K. Wu, "Optimal design of compact millimetre-wave SIW circular cavity filters," *Electron. Lett.*, Vol. 15, No. 19, 1068–1069, 2005.
5. Tang, H. J., W. Hong, J. X. Chen, G. Q. Luo, and K. Wu, "Development of millimeter-wave planar diplexers based on complementary characters of dual-mode substrate integrated waveguide filters with circular and elliptic cavities," *IEEE Trans. Microw. Theory Tech.*, Vol. 55, No. 4, 776–782, 2007.
6. Lin, S., S. Yang, A. E. Fathy, and A. Elsherbini, "Development of a novel UWB vivaldi antenna array using SIW technology," *Progress In Electromagnetics Research*, PIER 90, 369–384, 2009.
7. Zhang, X.-C., Z.-Y. Yu, and J. Xu, "Novel band-pass substrate integrated waveguide (SIW) filter based on complementary split ring resonators (CSRRs)," *Progress In Electromagnetics Research*, PIER 72, 39–46, 2007.
8. Qiang, L., Y.-J. Zhao, Q. Sun, W. Zhao, and B. Liu, "A compact UWB HMSIW bandpass filter based on complementary split-ring resonators," *Progress In Electromagnetics Research C*, Vol. 11, 237–243, 2009.
9. Amendola, G., G. Angiulli, E. Arnieri, and L. Boccia, "Resonant frequencies of circular substrate integrated resonators," *IEEE Microwave and Wireless Components Lett.*, Vol. 18, No. 4, 2008.
10. Ranjkesh, N. and M. Shahabadi, "Loss mechanisms in SIW and MSIW," *Progress In Electromagnetics Research B*, Vol. 4, 299–309, 2008.
11. Angiulli, G., E. Arnieri, D. de Carlo, and G. Amendola, "Fast nonlinear eigenvalues analysis of arbitrarily shaped substrate integrated waveguide (SIW) resonators," *IEEE Transactions on Magnetics*, Vol. 45, 1412–1415, 2009.

12. Angiulli, G., "On the computation of nonlinear eigenvalues in electromagnetic problems," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 4, 527–532, 2007.
13. Angiulli, G., D. de Carlo, G. Amendola, E. Arnieri, and S. Costanzo, "Support vector regression machines to evaluate resonant frequencies of elliptic substrate integrated waveguide resonators," *Progress In Electromagnetics Research*, PIER 83, 107–118, 2008.
14. Angiulli, G., M. Cacciola, and M. Versaci, "Microwave devices and antennas modelling by support vector regression machines," *IEEE Transactions on Magnetics*, Vol. 43, 1589–1592, 2007.
15. Cristianini, N. and J. Shawe-Taylor, *An Introduction to Support Vector Machines*, Cambridge Univ. Press, Cambridge, U.K., 2000.
16. The Spider MATLAB Toolbox, Apr. 19, 2006. Available online: http://www.kyb.tuebingen.mpg.de/bs/people/spider/main.html.