The inverse Davenport problem in $\mathbb{F}_p^2$.

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It is quite elementary that given any sequence $(a_1, a_2, \ldots, a_p)$ consisting of p elements from the field $\mathbb{F}_p$ of residue classes modulo p, there has to exist some non–empty subsequence the sum of whose terms vanishes. Moreover, it is only slightly harder to see that the number p of terms required here for this to happen is indeed sharp and that all sequences of length $p-1$ not admitting a non–empty zero–sum subsequence are constant. The questions answered by these two assertions are usually called the direct and the inverse Davenport problem for $\mathbb{F}_p$, respectively.

In two dimensions, i.e. if one tries to replace $\mathbb{F}_p$ by $\mathbb{F}_p^2$, both questions become significantly harder. As to the direct problem, it was proved in 1969 independently by Kruyswijk and Olson that any sequence $(a_1, a_2, \ldots, a_{2p-1})$ consisting of $2p-1$ points from $\mathbb{F}_p^2$ possesses a non–empty subsequence the sum of whose term equals 0. Their proofs are both algebraic and nowadays their result can be regarded as an easy consequence of Alon's Combinatorial Nullstellensatz. The corresponding inverse problem to classify all sequences of length $2p-2$ that do not have a zero–sum subsequence turned out to be much harder and although the correct answer was already conjectured in the 1970s it was proved only in 2009. In the talk we will try to explain the main ideas involved in this proof.