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Tribute to André Lichnerowicz (1915-1998)

Thirty years ago: *Annals of Physics*, **111**. Deformation quantization was born. This sequence of two articles [BBFLS 1978] had been preceded by several papers written by Daniel Sternheimer, who has just addressed you[1], by Moshé Flato whose memory Daniel has just recalled and whose personality and work in mathematics and physics were so remarkable, and by André Lichnerowicz.

Who was Lichnerowicz? To his students, he was "Lichné". To his friends, he was "André". As a mathematician, mathematical physicist, reformer of the French educational system and its mathematical curriculum, and as a philosopher, he was known to the public as "Lichnerowicz", a man of vast culture, an affable person, a great scientist.

There are many ways to approach André Lichnerowicz. One would be to read all his more than three hundred and sixty articles and books that have been reviewed in *Mathematical Reviews* (MathSciNet), or only read his personal choice among them, those that were published by the Éditions Hermann in 1982 as a 633-page book, *Choix d'œuvres mathématiques* [L 1982]. You can read the summaries of his work up to 1986 that were published by Yvonne Choquet-Bruhat, Marcel Berger and Charles-Michel Marle in the proceedings of a conference held in his honor on the occasion of his seventieth birthday [PQG 1988], or his portrait and an interview which appeared in a handsome, illustrated volume describing the careers of twenty-eight of

[1] This tribute was preceded by tributes to Stanisław Zakrzewski, founder of the series of international conferences on Poisson geometry, who died in April 1998 (by Alan Weinstein), to Paulette Libermann on the first anniversary of her death (by Charles-Michel Marle), and to Moshé Flato, co-author and friend of Lichnerowicz, who died in November 1998 (by Daniel Sternheimer). This tribute was followed by the award of the "André Lichnerowicz prize in Poisson Geometry" to two prominent young mathematicians. A French version of this tribute and of an announcement of the prize is to be found in the *Gazette des Mathématiciens*, October 2008.

the most important French scientists of the twentieth century, *Hommes de Science* [HdeS 1990]. Finally, and sadly, scientific obituaries appeared in the *Gazette des Mathématiciens* [Gazette 1999] and they were quickly translated for the *Notices of the American Mathematical Society* [Notices 1999] because Lichnerowicz was as famous outside France as he was within it. Other obituaries appeared in the *Journal of Geometry and Physics* of which he had been one of the founders, and in many other journals.

Lichnerowicz had an exceptional career. He was a student from 1933 to 1936 at the École Normale Supérieure in Paris where he studied under Élie Cartan, who had a lasting influence on his mathematics. He completed his thesis, written under the direction of Georges Darmois, in 1939 and was named professor of mechanics at the University of Strasbourg in 1941. Because of the war, the faculty of the University of Strasbourg had already moved to Clermont-Ferrand in order to avoid functioning under the German occupation. However in 1943, the Germans occupied Clermont-Ferrand as well, and there was a wave of arrests in which Lichnerowicz was taken, but he was fortunate enough, or daring enough, to escape. In those days, he did what he could to help those who were in mortal danger, in particular Jewish colleagues[2]. After the Liberation, the University of Strasbourg returned to Strasbourg. In 1949, he was named professor at the University of Paris, and then in 1952 he was elected to a chair at the Collège de France, the most prestigious position in French higher education[3].

When Lichnerowicz was elected to the Académie des Sciences de Paris – he was only 48, exceptionally young for a member of the Académie in those days – his students, as was customary, collected money to offer him his Academician's sword. (The sword is the only part of the Academician's very elaborate uniform which reflects his or her personality and accomplishments.) But two years later, for his fiftieth birthday, they contributed nearly as much to offer him something more to his taste, ... a pipe! Indeed we could not imagine him without his pipe at any time ... except during his lectures when he would fill the blackboard with equations in his dense handwriting,

[2] In his autobiography [S 1997, p. 200], Laurent Schwartz tells that one day Lichnerowicz contrived to be in a police station and, while an officer was inattentive, he managed to borrow a stamp and apply it to a false identity card which he, himself, did not need, but which could save the life of a colleague or a student. To survive the war years in France with some honor was itself a great achievement.

[3] A chair in mathematics was established by François I at the foundation of the Collège in 1531!

equations almost always comprising many tensorial indices. It is a fact that he can be seen in every photograph ... with his pipe.

Lichnerowicz's work in general relativity began with his thesis in which he gave the first global treatment of Einstein's theory of general relativity and determined necessary and sufficient conditions for a metric of hyperbolic signature on a differentiable manifold to be a global solution for the Einstein equations. He proved that there cannot exist any gravitational solitons; he established the "Lichnerowicz equation", an elliptic semi-linear equation used in the solution of the constraint equations to be satisfied by the inital conditions for Einstein's equations. He pursued this area of study throughout his research career. "Differential geometry and global analysis on manifolds", "the relations between mathematics and physics", "the mathematical treatment of Einstein's theory of gravitation", this is how he, himself, described his main interests and achievement in the interview published in *Hommes de Science* [HdeS 1990].

His work in Riemannian geometry remains particularly important. He was among the first geometers to establish a relation between the spectrum of the Laplacian and the curvature of the metric; he proved the now classical equivalence of the various definitions of Kähler manifolds; he showed, together with Armand Borel, that the restricted holonomy group of a Riemannian manifold is compact, and of course many other important results.

In the early sixties, Lichnerowicz established Cartan's and Weyl's theory of spinors in a rigorous differential geometric framework, on a pseudo-Riemannian manifold with a hyperbolic (Lorentzian) metric. Using this geometric approach in his courses at the Collège de France in 1962-1964, he developed Dirac's theory of the electron and that of Rarita-Schwinger for spin $\frac{3}{2}$, and then the Petiau-Duffin-Kemmer theory as well as the theory of the CPT transformations, while also in 1963 he published the landmark 'Note aux *Comptes rendus*' of the Académie on harmonic spinors [L 1963], in which he proved that, for any spinor field, ψ,

$$\Delta\psi = -\nabla^\rho\nabla_\rho\psi + \frac{1}{4}R\psi \ ,$$

where $\Delta = P^2$ is the Laplacian on spinors, the square of the Dirac operator P, ∇ is the covariant derivative, and R is the scalar curvature. And he continued working on spinors to his last days.

At the beginning of the seventies, Lichnerowicz's interest turned to the geometric theory of dynamical systems. Symplectic geometry had been studied for some time[4]. In January 1973, a conference, "Geometria simplettica e fisica matematica", was held in Rome that was, I believe, the first international meeting on this topic. As a young researcher I attended the congress, and heard and met many of the founders of symplectic geometry among whom were Jean Leray, Irving Segal, Bertram Kostant, Shlomo Sternberg, Włodzimierz Tulczyjew, Jean-Marie Souriau as well as the young Alan Weinstein and Jerry Marsden. And Lichnerowicz was one of the organizers of the meeting and delivered the opening lecture.

The main reason that we pay tribute to Lichnerowicz's memory here, today, at this conference on Poisson geometry, is that he founded it. This was a few years before the publication of the deformation quantification paper I recalled at the beginning of this tribute.

His son, Jérôme Lichnerowicz, speaking of his father's collaboration with Moshé Flato, said: "There was no master and no student but an incredible synergy between friends. I saw Moshé encourage André when, ageing, he doubted his own strength," and he added: "I heard Moshé tell me: 'It is unbelievable, he [Lichnerowicz] had an arid period, but now he is back doing mathematics as before!'." [CMF 2000] Starting in 1974, working with Moshé Flato and Daniel Sternheimer, Lichnerowicz formulated the definition of a Poisson manifold in terms of a bivector, *i.e.*, the contravariant 2-tensor advocated by Lie, Carathéodory and Tulczyjew, which is the counterpart of the 2-form of symplectic geometry. In his article published in *Topics in Differential Geometry* [L 1976] he defined the canonical manifolds and one can already find in that paper a formula for the bracket of 1-forms associated to a Poisson bracket of functions, although still only for exact forms,

$$[df, dg] = d\{f, g\} \ .$$

(Later he showed that the canonical manifolds are those Poisson manifolds whose symplectic foliation is everywhere of co-dimension one.) In his 1977 article in the *Journal of Differential Geometry*, "Les variétés de Poisson et leurs algèbres de Lie associées" [L 1977], Lichnerowicz introduced the cohomology operator that is now called the "Poisson cohomology operator" but

[4] As Hermann Weyl explained at the beginning of Chapter VI of his *The Classical Groups* [W 1939], he had coined the adjective "symplectic" after the Greek as an alternative to the adjective "complex".

really should be called the “Lichnerowicz-Poisson cohomology operator”, a profound discovery. We read there as well as in [BFFLS, 1978] that, in the particular case of a symplectic manifold,

$$\mu([G, A]) = d\mu(A) \ ,$$

where A is a field of multivectors. (The notations are G for the Poisson bivector and μ for the prolongation to multivectors of the isomorphism from the tangent bundle to the cotangent bundle defined by the symplectic form, the bracket is the Schouten-Nijenhuis bracket, and d is the de Rham differential.) This formula, which we rewrite in a more familiar notation,

$$\omega^\flat([\pi, A]) = d(\omega^\flat(A)) \qquad \text{or} \qquad \pi^\sharp(d\alpha) = d_\pi(\pi^\sharp\alpha)$$

(here G is replaced by π, and μ by $\omega^\flat$, with inverse $\pi^\sharp$, while α is a differntial form and d_π is the Lichnerowicz-Poisson differential, $d_\pi = [\pi, \cdot]$, acting on multivectors) is the precursor of the chain map property of the Poisson map, mapping the de Rham complex to the Lichnerowicz-Poisson complex, and, more generally, this article is the point of departure for the great development of Poisson geometry that we have witnessed and in which we are participating here. Together with his earlier articles written jointly with Flato and Sternheimer [FLS 1974, 1975, 1976] and with the article in the *Annals of Physics* [BBFLS 1978], solving quantization problems by a deformation of the commutative multiplication of the classical observables when given a Poisson structure, this article established the foundation of what has become a vast field of mathematical research.

It was a privilege for Lichnerowicz's many doctoral students, of whom I was one in the late 1960s, to be received by him in his small office under the roof of the Collège de France, or in the study of his apartment on the Avenue Paul Appell, on the southern edge of Paris. Surrounded by collections of journals and piles of papers, Lichné, with his pipe, would offer encouragement and invaluable hints as to how to make progress on a difficult research problem. I knew then, we all knew, that we were talking to a great mathematician. But I did not even guess that I was talking to the creator of a theory which would develop into a field in its own right, one with ramifications in a very large number of areas of mathematics and physics.

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