7. $\boxed{\text{Talk 4}}$ Small topological degree.

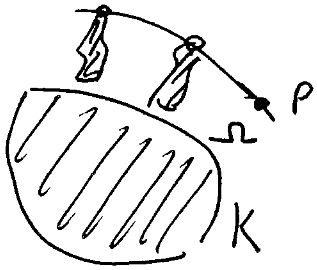
$F : \mathbb{C}^2 \to \mathbb{C}^2 \quad e < \lambda$.

$\boxed{\text{Thm}}$ $\exists\ \pi : X \to \mathbb{P}^2$ iso above $\mathbb{C}^2$.

$p \in \pi^{-1} L_\infty \quad N \gg 1$

1) $F^N(\pi^{-1}(L_\infty) \setminus I(F^N)) = p$.

2) F is hol at p and $(\mathrm{Crit}(F), p) \subseteq \pi^{-1}(L_\infty)$



Ω = basin of attraction of p in $\mathbb{C}^2$

either $z \in \Omega$ and $\|F^n(z)\| \simeq c^{\lambda^n}$

or $\|F^n(z)\| \leq c^{(e+\varepsilon)^n}$.

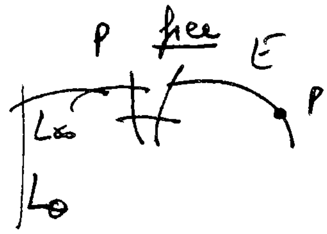
→ $e<\lambda \Rightarrow F$ not a skew product hence $dg(F^n) \simeq \lambda^n$. [talk 2]

$\Rightarrow e<\lambda^2$! hence $dg(F^n) = c\lambda^n + O(\sqrt{e}^n)$ [talk 3].

→ $dg(F^n) = (F^n)^\# L_\infty \cdot L_\infty$ in $NS(X)$ linear recurrence relation.

$\quad \| \ (F^\#)^n (F^N)^\#$ as F is AS.

→ proof that $z \in \Omega \Rightarrow \|F^n(z)\| \simeq c^{\lambda^n}$.



$L_\infty = \sum a_E E \qquad a_E > 0$

$\lambda^n \sim dg(F^{n+N}) = f^{n+N}(L_0) \cdot L_\infty$

$= a_E\, f^n(L_0^N) \cdot E. \quad n \geq 0$

$$F(z,w) = \left(z^k(1+\alpha), \; \cancel{k}_2\right)$$

$$= \left(z^k, \; \lambda w z^\ell_{(1+\alpha)} + \varphi(z)\right)$$

$$\underset{u}{F^n(C_N) \cdot E} = \cancel{\ldots}$$

$\cancel{\ldots (0,t)}$

$\cancel{\ldots}$

$t \to (\phi_1(t), \; \phi_2(t))$ parametrizes C_N

$$\downarrow F^N$$

$$\mathrm{ord}_0\left(E^m_0 \, \phi_1(t)\right) = k^n \, \mathrm{cte}.$$

$\underline{k = \lambda}$

$\square$

<u>Meaning</u> = important fact is that if you stay in boxes near $Z(F)$ you cannot go to ∞ very fast.

2/

Strategy of proof. basic strategy = using action F_* on L^2
get convergence of all valuations to a *fixed* one

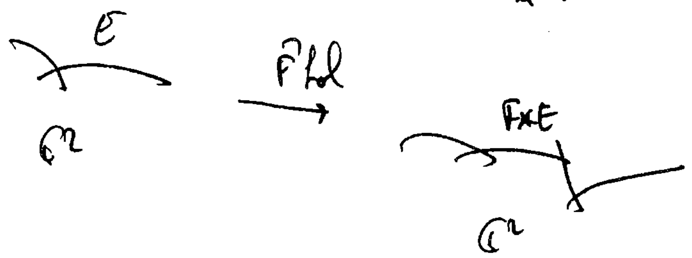
- Recall
  - $\mathcal{V}_0$ $\mathbb{R}$-tree.
  - $v = \mathrm{ord}_E$ divisorial $\to Z_E$ $\quad \alpha(v) = \frac{1}{b_E^2} Z_E^2 \geq 0$.

    & parametrization of $\mathcal{V}_0$.

- view $Z_E \in$ C-NS

(Lep) $F_* Z_v = Z_{F_* v}$ $\quad v$ divisorial.

proof $\qquad F_* Z_v \cdot E' = Z_v \cdot F^* E'$.



□.

(Prop) $e < \lambda^2$

$\forall v \in \mathcal{V}_0$ div. either $F_* v = t v$

or $\frac{1}{\lambda^n} F_*^n v \to v_*$ $\in \mathcal{V}_0$

proof.

- action on L^2: $F_* \theta_+ = \lambda \theta_+$

  $Z \in L^2 \qquad Z^2 > 0 \qquad \frac{1}{\lambda^n} F_*^n Z \to c\, \theta_+$.

$$Z = \mathcal{L} \qquad \frac{1}{\lambda^n} F_*^n \mathcal{L} = \frac{1}{\lambda^n} Z_{F_*^n(-d_3)} = \left.\underbrace{\frac{d_3(F^n)}{\lambda^n}}_{\text{cte}} Z_{F_*^n(-d_3)}\right\} \to \theta_+$$

$\theta_f = Z_v$ for some $v \in \mathcal{V}_0$

because $\theta_f \cdot C_\ell = \lim Z_{v_n} \cdot C_\ell$. $\square$.

Prop $X \xrightarrow{\pi} \mathbb{P}^2$ ~~$\forall$ all E $\cdots \in \mathcal{V}_0$~~.

$p \in E \quad \alpha(E) > 0 \Rightarrow \alpha(E_p) \geq 0$.

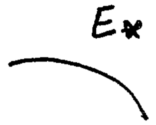
proof.

~~p ... a free point~~.

- p free point $\rightsquigarrow$ $\alpha(E) = Z_E^2 \in \mathbb{N}^*$.

$\alpha(E_p) = Z_E^2 - 1$ $\square$.

<u>the proof</u> assume $v_* \in \mathcal{V}_{qm}$.

1. 

$F_* v_* = \lambda v_*$ $\Rightarrow$ $\lambda \geq e$.
Hence $v_* \in \mathcal{V}_{qm}$ is irrational.

2. smallest sequence of pt blow ups.

$$X_{n+1} \xrightarrow{\mu_n} X_n$$

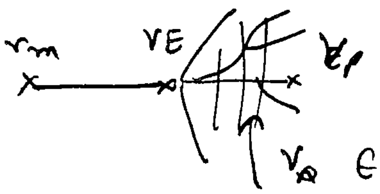
for all $E \subseteq X_n$ $\alpha(\mathrm{ord}_E) \geq 0$.

<u>proof</u> ~~induction~~ induction.

+ p_n = intersection of two divisors E E'
  if $v_E \geq v_{E'}$ (or the converse) $\alpha(v_{E'}) \geq \alpha(v_E) \geq 0$

  apply prop

+ p_n = free pt on E. with $\alpha(E) = 0$.



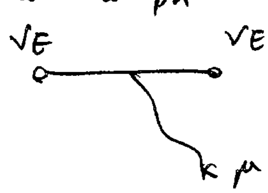
□.

3. Pick $n \gg 0$ $\mathrm{Crit}(F) \simeq \mathbb{C}^2$ does not go through p_n.

<u>proof</u> at a branch for $n \gg 0$ $\Rightarrow$ always blow-up at a free pt $\alpha_n = \lambda - \frac{n}{\beta^2} \to -\infty$. □.

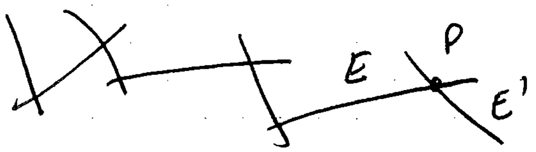
4. $U_n \subseteq \mathcal{V}$ valuations centered at p_n

ie 



$F_\bullet^2(U_n) \subseteq U_n$ for some n.

Fix this $n \geq n_0$.



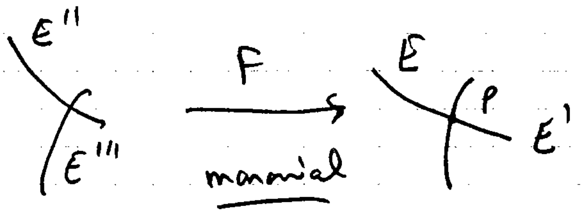
a) F hol. at p because of ①)

b) F^N any curve $= p$ ~~because~~ of

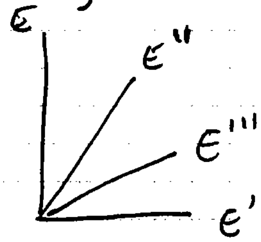
$\times\ \alpha(E) \geq 0 \rightsquigarrow$ if $\frac{1}{\lambda^n} F^n_* z_E \not\to z_0$ then

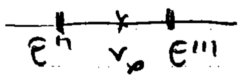
$F_* z_E = z_E \Rightarrow e \geq 1$.

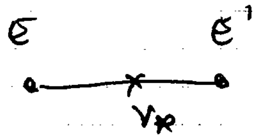
Fact ①)



locally $F = (z^a w^b, z^c w^d)$







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