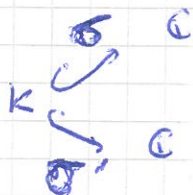


Suppose two embeddings define the same norm



then the ~~map~~ identity map on K induces a field automorphism on the complex numbers

$$\hat{K}_\sigma \xrightarrow{\varphi} \hat{K}_{\sigma'} \text{ which is continuous (bottic)$$

- \Rightarrow if $\hat{K}_\sigma = \mathbb{R}$ then $\hat{K}_{\sigma'} = \mathbb{R}$ and $\varphi = \text{id}$
 if $\hat{K}_\sigma = \mathbb{C}$ then $\hat{K}_{\sigma'} = \mathbb{C}$ and $\varphi = \text{id or } \overline{}$.

$$M_{K,p} = \{ \text{l.i. multiplicative norms on } K, \text{ l.i. } \ominus = \text{l.i. } \rho \} \quad \rho \text{ prime}$$

use same analysis $P = \prod_{i=1}^r P_i$ P_i irreducible over \mathbb{Q}_p
 $\deg(P_i) = n_i$.

Thm $M_{K,p}$ has exactly 2^r elements.
 For each $v \in M_{K,p}$ $\exists n_v = [K_v : \mathbb{Q}_p]$
 then $\prod_{v \in M_{K,p}} |x|_v^{n_v} = |N_{K/\mathbb{Q}}(x)|_p \quad \forall x \in K$.

obs: $[K_v : \mathbb{Q}_p]$ can be arbitrary large.

proof. $\mathbb{Q} \subseteq \mathbb{Q}_p \subseteq \mathbb{Q}_p^{\text{abs-closed}} \subseteq \mathbb{C}_p$. zeros of $f = \{x_1, \dots, x_n\}$
 $K \subseteq K \subseteq \mathbb{C}_p$

claim: \hat{K}/\mathbb{Q}_p is a finite extension

pick any inclusion $\hat{K} \subseteq \mathbb{Q}_p^{\text{abs}}$ respecting \mathbb{Q}_p = maybe several but they are all isometric since \hat{K} admits a unique metric

$$\Rightarrow \exists \sigma: K \hookrightarrow \mathbb{C}_p \quad |x| = |\sigma(x)|_p \quad \sigma = \sigma_i \text{ for some } i \quad \sigma_i(\tau) = x_i$$

* if $\sigma_i, \sigma_j: K \hookrightarrow \mathbb{C}_p \quad |\sigma_i(x)|_p = |\sigma_j(x)|_p$ then $\hat{K}_i \cong \hat{K}_j$
 isometric field automorphism identity on \mathbb{Q}_p

$$\Rightarrow \exists \varphi \in \text{Gal}(\mathbb{Q}_p^{\text{abs}}/\mathbb{Q}_p) \quad \sigma_i = \varphi \sigma_j$$

$\Leftrightarrow x_i \& x_j$ are zeros of the same irreducible factor of P over \mathbb{Q}_p - TVSP