Curriculum vitae of Yann Brenier, 2012

Born January 1st 1957, Saint-Chamond, France, French citizen and resident.

Centre de mathématiques Laurent Schwartz,
Ecole Polytechnique, FR-91128 Palaiseau, France
(new address, since sept. 1, 2012)

brenieratmath.polytechnique.fr

website : http : //www.math.polytechnique.fr/ brenier/

Education and positions

2012– Senior researcher (Directeur de recherches) CNRS,
Centre de mathématiques Laurent Schwartz, Ecole Polytechnique, FR-91128 Palaiseau, France

2000-2012 Senior researcher (Directeur de recherches) CNRS, Université de Nice (permanent member of CNRS since 2005, formerly on leave from U. Paris 6).

1990-2005 Professor, Université Paris 6 (including 7 years as full-time professor at Ecole Normale Supérieure, rue d'Ulm, Paris, 1990-1997).

1986-1990, Senior researcher (Directeur de recherches), INRIA, Rocquencourt.

1985-1986 J.R. Hedrick Assistant Professor, University of California (UCLA).

Doctorat ès Sciences, 1986, Université Paris-Dauphine, thesis committee: P.-A. Raviart, C. Bardos, G. Chavent, B. Engquist, J.-M. Lasry, M. Schatzman, L. Tartar.

1979-1985, junior researcher (chercheur), INRIA, Rocquencourt (including 15 months of national service, as researcher at the IIMAS-Universidad Nacional Autonoma de México, Mexico city)

Doctorat de 3ème cycle (PhD), Paris-Dauphine, 1982, advisor : Guy Chavent.

Distinctions

Junior member of the Institut Universitaire de France (1996-2000),
Invited sectional lecture at ICM 2002, Beijing,
Invited plenary lecture at ICIAM 2003, Sydney,
Prix Petit-d'Ormoy of the French Academy of Sciences 2005.

Other distinctions and special lectures

Member of the Institute for Advanced Study 1992.
Prix des annales de l'IHP (with L. Corrias) 1999.
Charles Amick lectures, University of Chicago, may 2003.
Aziz lectures, University of Maryland, may 2006.
Lipschitz lectures, Universität Bonn, june 2007.
Nachdiplomvorlesung, ETH, Zurich, fall 2009.

Main research fields

PDEs and numerical analysis, Mathematical problems of fluid mechanics and plasma physics, Optimal transport theory.

PhD Students

Jean-David BENAMOU, PhD completion : November 1992, Senior researcher, INRIA Rocquencourt, since 2000.
Lucilla CORRIAS, PhD completion : June 1995, Maitre de conferences, Université d'Evry, since 1996.
Michel ROESCH, PhD completion : October 1995, Member of the Corps des Mines, since 1994, currently at PSA Peugeot Citroen
Emmanuel GRENIER, PhD completion : October 1995, Full Professor, Ecole Normale Supérieure de Lyon, since 1998.
Alexis VASSEUR, PhD completion : 1998, Full Professor, UT Austin.
Marjolaine PUEL, PhD completion : May 2001, Associate Professor, Université de Nice, since 2012.
Grégoire LOEPER, PhD completion : December 2003, BNP-Paribas London (on leave from Université de Lyon)

Visits (one to three months) since 2002

Cambridge, Newton Institute, mar. 2003 (Program on Hyperbolic PDEs),

SNS Pisa, Centro De Giorgi, apr. 2004 (Program on Phase Space Analysis of Partial Differential Equations),

EPFL, Bernoulli center, aug. 2004 (Program on Geometric Mechanics and Its Applications),

EPFL, Bernoulli center, sep. 2006 (Program on Asymptotic Behavior in Fluid Mechanics),

SNS Pisa, Centro De Giorgi, nov. 2006 (Program on Calculus of Variations and Partial Differential Equations),

Universität Wien, aug-sep. 2007,

Universität Bonn, apr-jun. 2007 (Lipschitz lectures),

UCLA, may-jun. 2008 (IPAM program on "optimal transport"),

Senter for matematikk for anvendelser (CMA, Special Year in Nonlinear Partial Differential Equations), sep. 2008, Oslo,

ETH, oct-dec. 2009 (Nachdiplomvorlesung),

Cambridge, Newton Institute (Program on Partial Differential Equations in Kinetic Theories), aug-sep. 2010,

Universitá degli Studi dell'Aquila, oct-nov. 2010.

SISSA, Trieste, march-apr. 2012.

Invited Conference and Workshops -since 2002

International Congress of Mathematicians, sectional lecture, Beijing 2002.

Plenary talks in large international conferences with parallel sessions, since 2002

International Congress on Industrial and Applied Mathematics ICIAM, Sydney, 7-11 july 2003.

SIAM Conference on Analysis of PDEs, 7-10 dec. 2009, Miami, USA.

Equadiff 2011 (plenary speaker), 1-5 aug. 2011, Loughborough, UK

Oberwolfach Invitations (with talks) since 2002

Thermodynamische Materialtheorien (15-21 dec 2002, Hunter-Müller-Truskinovsky),

Partielle Differentialgleichungen (03-09 august 2003, Kuwert-Otto-Simon),

Hyperbolic Conservation laws (04-10 april 2004, Dafermos-Kröner-LeVeque),

Calculus of Variations (13-19 june 2004, dal Maso-Friesecke-Rivière),

Calculus of Variations (9-15 july 2006, Alberti-McCann-Rivière),

Atmosphere-Ocean Science (20-26 aug 2006, Buehler-Majda-Klein),

Classical and Quantum Mechanical Models of Many-Particle Systems (3-9 dec 2006, Arnold-Cercignani-Desvillettes),

PDEs (Ilmanen, Schaetzle, Trudinger) 22-28 jul. 2007,

Material Theories (DeSimone, Luckhaus, Truskinovsky) 16-22 dec. 2007,

Hyperbolic conservation laws (Dafermos, Kroener, LeVeque) 7-13 dec 2008,

Mathematical Aspects of Hydrodynamics (Seregin, Sverak) 19-25 jul 2009,

Material Theories (DeSimone, Luckhaus, Truskinovsky) 12-19 dec. 2009,

Atmosphere-Ocean-Science (Majda, Stevens, Klein), 8-14 aug 2010.

Classical and quantum systems of paricles (Arnold, Carlen, Desvillettes) 6-10 dec. 2010.

Variational Methods for Evolution (Mielke, Otto, Savaré, Stefanelli) 4-10 dec. 2011.

Interplay of Analysis and Probability in Physics (König, Mörters, Peletier, Zimmer), 22-28 jan 2012,

Mathematical Aspects of Hydrodynamics (Constantin, Fridelander, Seregin, Titi), 12-18 Aug 2012.

Conferences and workshops, since 2002

International Conference on Scientific Computing and Partial Differential Equations (on the Occasion of Stanley Osher's 60th birthday), Hong Kong, 12-15 dec 2002.

Optimal Transportation and Nonlinear Dynamics Workshop, PIMS, Vancouver, August 11-1, 2003,

Workshop on Calculus of Variations: Geometric Problems, Superconductivity, and Material Microstructures Fields Institute, Toronto, August 25-29, 2003,

Optimal Transport Theory and Applications, SNS, Pisa, 9-12 october 2003,

Workshop on Kinetic Theory, Fields Institute, Toronto, March 29-April 2, 2004,

International Workshop on Nonlinear Waves (on the Occasion of George Papanicolaou's 60th birthday), Hong Kong, June 1-4 2004.

Numerical Methods for Viscosity Solutions and Applications Rome, September, 6-8, 2004,

The Sixth International Workshop on Mathematical Aspects of Fluid and Plasma Dynamics, Kyoto, September 19-23, 2004.

MathGeo 04, New Trends in Mathematical and Numerical Methods for Geosciences Direct and Inverse Problems (A conference honoring Guy Chavent), Inria-Rocquencourt, France December 9-10, 2004.

Issues on computational transport in meso and nano scales, Institute for Computational Engineering and Sciences (ICES), U. of Texas, Austin, March 4-5, 2005.

4th International Workshop on Kinetic Theory and Applications, Karlstad University, Sweden, June 12-14 2005.

Conference in honor of Björn Engquist's 60th birthday, KTH, Stockholm, June 16-17 2005.

International Forum on Multiscale Methods and PDEs, Institute for Pure and Applied Mathematics, UCLA, Los Angeles, August 26-27 2005.

Optimal Mass Transport and its Applications, MSRI, Berkeley, November 14-18 2005.

Geometric and Nonlinear Analysis , BIRS, Banff, Aug 12-17 2006,

Fluides en rotation en géophysique, Centre Bernoulli, EPFL, Lausanne, Switzerland, 19-22 Sept 2006.

Optimal transport: theory and applications, de Giorgi Center, Pisa, 14-18 Nov 2006.

Clifford Conference, Nonlinear PDEs: Analysis, Numerics, and Applications, 21-24 mar. 2007, New-Orleans, USA,

INdAM International workshop on Nonlinear Hyperbolic Problems, 28 may-1 jun. 2007, Rome,

Euler Equations: 250 Years, 19-22 jun. 2007, Aussois, France,

Des EDP au calcul scientifique, Congrès en l'honneur de Luc Tartar, 2-6 jul. 2007, Paris,

Workshop Optimal Transportation, and Applications to Geophysics and Geometry, 16-20 jul. 2007, ICMS, Edinburgh, Conference XX CEDyA, 24-28 sep. 2007, Sevilla,

Numerics and Dynamics for optimal transport, 14-18 avril 2008, IPAM, UCLA, Los Angeles,

IPAM program on Optimal Transport: Culminating Workshop at Lake Arrowhead 8-13 jun. 2008, Lake Arrowhead, USA,

CIMPA School on Nonlinear Analysis and Geometric PDEs, 15-24 jun. 2008, Tsaghkadzor, Armenia,

Geometric Analysis, Elasticity and PDE (60th birthday of John Ball), 23-27 jun. 2008, Edinburgh,

Nonlinear PDEs at IMPA, 4-8 aug. 2008, Rio de Janeiro,

International Conference on Contemporary Applied Mathematics, (dedicated to Prof. Andrew Majda on the occasion of his 60th birthday), 19-23 jan. 2009, Fudan University, Shanghai,

Workshop: At the interface of dynamical and statistical cosmology and transport optimization, 22-26 mar. 2009,Technion, Haifa,

Workshop on the mathematics of weather and climate prediction (mini-course) 30 mar-3 apr. 2009, Met Office, Exeter, UK,

Conference on optimal transportation : Theory and applications, Institut Fourier, 28 jun-3 jul 2009, Grenoble, France,

Workshop on Regularity problems in hydrodynamics, 3-7 aug 2009, PIMS, Vancouver,

Asymptotics in Complex Systems, INDAM conference, 28 sep-2 oct 2009, Corinaldo, Italiy,

Worshop on Hyperbolic Conservation Laws and Fluid Mechanics, 15-17 feb. 2010, Parma,

Geometric Evolutions and Gradient Flow, 5-7 may 2010, Metz, France,

International Conference on Applied Mathematics, 7 - 11 june, 2010, City

University, Hong-Kong,

Workshop on Optimal transport and Kinetics Applied to Socio-Economics conference, 1-3 sep 2010, Toulouse 1, France,

Workshop on Geometric Evolutions and Minimal Surfaces in Lorentzian Manifolds, 7-10 sep. 2010, Centro De Giorgi, Pisa,

Workshop on Fluid-Kinetic Modelling in Biology, Physics and Engineering, 6-10 sep. 2010, Newton Institute, Cambridge,

ERC Workshop on Optimal Transportation and Applications. 12-16 oct. 2010, Centro De Georgi, Pisa.

South West Regional PDE Winter School, 10-11 feb. 2011, Bristol, UK.

ICMS conference: Dynamical systems and classical mechanics: a conference in celebration of Vladimir Arnold, 3-7 oct 2011, Edimbourg, UK (organizers: Eliasson, Kuksin, Toland).

IAS Workshop on Symplectic Dynamics II (IAS special year on symplectic dynamics) 12-16 mar. 2012 (organizers: Hofer, Mather).

Optimal Transport (to) Orsay, 18-22 juin 2012, Orsay, (organizers: Bolley, Carlier, Louet, Santambrogio)

Main responsabilities (since 2002)

Chairmanships

Since September 2008 until August 2012, I have been the chairman of the CNRS hiring, evaluation and promotion committee for mathematics (prësident de la section de mathématiques du comité national du CNRS). This committee is made of about 20 members and has a steering group of 5 people. This committee is in charge of evaluating and promoting about 400 CNRS researchers in mathematics. Each year, more than 20 people are hired (from outside) by CNRS according to our suggestions (the final decision is made by the direction of CNRS, which usually follows our advices).

Previously, I was the director (01-01-2004/01-01-2008) of the Fedération Wolfgang Doeblin", an interdisciplinary CNRS joint venture of 3 research departments in Nice:
-Laboratoire Jean-Alexandre Dieudonné (department of mathematics),
-Institut non-linéaire de Nice (physics and nonlinear sciences),
-Laboratoire Cassiopée (fluid mechanics, turbulence and cosmology) of the observatoire de la côte-d'azur.
This organism was of small size and limited budget, but played a substantial role in gathering mathematicians and physicists, in particular in the maths department.

Editorial boards

Archive of Rational Mechanics and Analysis (2000-2010), Kinetic and related Moidels (since 2008), Journal of Geometric Mechanics since 2012.

List of papers 2000-2012

(and selected publications prior to 2000)

See also talks, preprints, numerics etc... on http://math.unice.fr/ brenier

References

[1] avec W. Gangbo, G. Savaré, M. Westdickenberg, Sticky particle dynamics with interactions to appear in J. Maths pures et appliquées.

[2] Remarks on the Minimizing Geodesic Problem in Inviscid Incompressible Fluid Mechanics, arXiv:1011.1104, to appear in Calc. Variations and PDEs.

bibitemHAL A modified least action principle allowing mass concentrations for the early universe reconstruction problem, Confluentes Mathematici 3 (2011) 361-385.

[3] avec C. De Lellis, L. Szekelyhidi, Weak-Strong Uniqueness for Measure-Valued Solutions, Comm. Math. Physics, Comm. Math. Phys. 305 (2011) 351-361.

[4] avec F. Otto, Ch. Seis, Upper bounds on coarsening rates in demixing binary viscous liquids, SIAM J. Math. Anal. 43 (2011) 114-134.

[5] Remarks on the Minimizing Geodesic Problem in Inviscid Incompressible Fluid Mechanics, arXiv:1011.1104

[6] Hilbertian approaches to some nonlinear conservation laws, accepted (2010) for publication in the Contemporary Mathematics volume based on the 2008-2009 Special Year in Nonlinear Partial Differential Equations, edited by Helge Holden and Kenneth H. Karlsen.

[7] Hidden convexity in some nonlinear PDEs from geometry and physics, Journal of Convex Analysis 17 (2010), No. 3-4, 945–959

[8] On the hydrostatic and Darcy limits of the convective Navier-Stokes equations, Chin. Ann. Math. Ser. B 30 (2009), no. 6, 683696,

[9] with M.J. Cullen, Rigorous derivation of the x-z semigeostrophic equations. Commun. Math. Sci. 7 (2009), no. 3, 779784.

[10] Optimal Transport, Convection, Magnetic Relaxation and Generalized Boussinesq equations, J. Nonlinear Sci. 19 (2009), no. 5, 547570.

[11] L2 formulation of multidimensional scalar conservation laws, Archive Rat. Mech. Analysis 193 (2009) 1-19.

[12] with Nicolas Besse, Florent Berthelin, Pierre Bertrand, The multi-water-bag equations for collisionless kinetic modeling, Kinetic and Related Models, 2 (2009) 39-80.

[13] Generalized solutions and hydrostatic approximation of the Euler equations, Physica D 237 (2008) 1982-1988.

[14] Non relativistic strings may be approximated by relativistic strings, Methods and Applications of Analysis 12 (2005) 153-16.

[15] Hyperbolic PDEs, kinetic formulation and geometric measure theory, European Congress of Mathematics, 555-560, Eur. Math. Soc., Zürich, 2005.

[16] Le système de Born-Infeld élargi: des ondes aux particules et aux cordes. Séminaire: Équations aux Dérivées Partielles. 2004-2005, Exp. No. VII, École Polytech., Palaiseau, 2005.

[17] with Wen-An Yong, Derivation of particle, string, and membrane motions from the Born-Infeld electromagnetism, J. Math. Phys. 46 (2005) 6, 062305.

[18] Solutions with concentration to the Riemann problem for the one-dimensional Chaplygin gas equations, J. Math. Fluid Mech. 7 (2005) suppl. 3, S326-S331.

[19] with François Bolley and Grégoire Loeper, Contractive metrics for scalar conservation laws, J. Hyperbolic Differ. Equ. 2 (2005) 91-107.

[20] Order preserving vibrating strings and applications to electrodynamics and magnetohydrodynamics, Methods Appl. Anal. 11 (2004) 515-532.

[21] with Grégoire Loeper, A geometric approximation to the Euler equations: the Vlasov-Monge-Ampère system, Geom. Funct. Anal. 14 (2004) 1182-1218.

[22] Hydrodynamic Structure of the augmented Born-Infeld equations, Arch. Ration. Mech. Anal. 172 (2004) 65-91.

[23] Deformations of 2D fluid motions using 3D Born-Infeld equations, Monatsh. Math. 142 (2004) 113-122.

[24] with Roberto Natalini and Marjolaine Puel, Relaxation of the incompressible Navier-Stokes equations, Proc. Amer. Math. Soc. 132 (2004) 1021-1028.

[25] with U. Frisch, M. Henon, G. Loeper, S. Matarrese, R. Mohayaee, A. Sobolevskii, Reconstruction of the early Universe as a convex optimization problem, Mon. Not. Roy. Astron. Soc. 346 (2003) 501-524.

[26] with Norbert Mauser and Marjolaine Puel, Derivation of e-MHD from the Vlasov-Maxwell system, Commun. Math. Sci. 1 (2003) 437-447.

[27] Remarks on the derivation of the hydrostatic Euler equations, Bull. Sci. Math. 127 (2003) 585-595.

[28] Harmonicity up to rearrangement and isothermal gas dynamics, Commun. Math. Sci. 1 (2003) 13-29.

[29] Topics on hydrodynamics and volume preserving maps, Handbook of mathematical fluid dynamics, Vol. II, 55-86, North-Holland, Amsterdam, 2003.

[30] Extended Monge-Kantorovich theory. Optimal transportation and applications, 91-121, Lecture Notes in Math., 1813, Springer, Berlin, 2003.

[31] with Wilfrid Gangbo, L^p approximation of maps by diffeomorphisms, Calc. Var. Partial Differential Equations 16 (2003), no. 2, 147-164.

[32] with Marjolaine Puel, Optimal multiphase transportation with prescribed momentum, ESAIM Control Optim. Calc. Var. 8 (2002), 287-343.

[33] Some geometric PDEs related to hydrodynamics and electrodynamics, Proceedings of the International Congress of Mathematicians, Vol. III (Beijing, 2002), 761–772, Higher Ed. Press, Beijing, 2002.

[34] with Jean-David Benamou, Mixed L^2-Wasserstein optimal mapping between prescribed density functions, J. Optim. Theory Appl. 111 (2001), no. 2, 255–271.

[35] Derivation of the Euler equations from a caricature of Coulomb interaction, Comm. Math. Physics. 212 (2000) 93-104.

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[37] with Doron Levy, Dissipative behavior of some fully non-linear KdV-type equations, Phys. D 137 (2000) 277-294.

[38] with Jean-David Benamou, A Computational Fluid Mechanics solution to the Monge-Kantorovich mass transfer problem, Numer. Math. 84 (2000) 375-393.

[39] with Bouchut, Cortes and Ripoll, A hierarchy of models for two-phase flows, J. Nonlinear Sci. 10 (2000) 639-660.

[40] Homogeneous hydrostatic flows with convex velocity profiles, Nonlinearity 12 (1999) 495 - 512.

[41] Minimal geodesics on groups of volume-preserving maps and generalized solutions of the Euler equations, Comm. Pure and Applied Maths, 52 (1999) 411-452.

[42] with Emmanuel Grenier, Sticky particles and scalar conservation laws, SIAM J. Numer. Anal. 35 (1998) 2317-2328.

[43] with Jean-David Benamou, Weak existence for the semigeostrophic equations formulated as a coupled Monge-Ampère transport problem, SIAM J. Appl. Math. 58 (1998) 1450-1461.

[44] with Lucilla Corrias, A kinetic formulation for multi-branch entropy solutions of scalar conservation laws, . Ann. Inst. H. Poincaré Anal. Non Linéaire 15 (1998) 169-190.

[45] A homogenized model for vortex sheets, Arch. Rational Mech. Anal. 138 (1997) 319-353.

[46] with G.-H. Cottet, Convergence of particle methods with random rezoning for the 2-D Euler and Navier-Stokes equations, (1995), SIAM J. of Num. Analysis 32 (1995) 1080-1097.

[47] The dual least Action Problem for an incompressible fluid, Archive for Rat. Mech. 122 (1993) 323-351.

[48] Polar factorization and monotone rearrangement of vector valued functions, Comm. Pure and Appl. Math. 64 (1991) 375-417.

[49] Une méthode particulaire pour les équations non linéaires de diffusion convection en dimension un. J. Comput. Appl. Math. 31 (1990), 35-56.

[50] The least Action Principle and the related concept of generalized flows for incompressible inviscid fluids, J. of the A.M.S, 2 (1989) 225-255.

[51] A combinatorial algorithm for the Euler equations of incompressible fluids, Comp. Meth. in Appl. Mech. and Engineering 75 (1989) 325-332.

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[53] with S.Osher, The one-sided Lipschitz condition for convex conservation laws, SIAM J. Num. Analysis 25 (1988) 8-28.

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[56] Résolution d'équations d'évolution quasilinéaires en dimension N d'espace à l'aide d'équations linéaires en dimension N+1, J.Diff.Equ. 50 (1983) 375-390.

[57] Une équation homologique avec contrainte, C. R. Acad. Sci. Paris 295 (1982) 103-106.

Description of some works since 2004

A modified least action principle to handle mass concentrations for the early universe reconstruction problem

This work is contained in [?].
Some years ago, with Uriel Frisch and collaborators [25], we adressed the reconstruction problem for the early universe (EUR), following PJE Peebles: right after the Big Bang, the density of matter in the universe is nearly uniform, with very tiny fluctuations that, in principle, should be predicted by quantum gravity theories. The EUR problem is about recovering these initial fluctuations from the present observation of the mass distribution in the universe. The model used in [25] for this purpose is just a (highly) simplified version of the full Einstein equations, namely the pressure-less gravitational Euler Poisson system, with time dependent coefficients taking into account Big Bang features. (This amounts to considering Newtonian gravitation in an Einstein de Sitter background.) We observed in [25] that this problem could be solved just by minimizing the total action of the matter integrated in (co-moving) time from the Big Bang up to now. Indeed, this action turns out to be strictly convex with respect to the density, momentum and gravitational fields. Unfortunately, the standard least action principle is unable to take into account mass concentration effects, which are unavoidable, as shown by physical evidence (the present distribution of matter is highly concentrated) and, more precisely, by the example of some special solutions discovered by Zeldovich in the 70s. In [?], we show that a modification of the action is possible which allows dynamical concentrations, at least for solutions depending only on one space variable. For this purpose, we substitute the fully nonlinear Monge-Ampere equation for the linear Poisson equation to model gravitation, which is the same for one space variable (we ignore whether or not this is a reasonable approximation for the full Einstein equations in higher dimensions). The resulting MAG model enjoys a least action principle with a very nice geometric interpretation, in which we can input mass concentration effects in a canonical way, based on the theory of gradient flows with convex potentials. (This is somewhat related to the concept of self-dual Lagrangians, as in Yang-Mills theory.) A fully discrete algorithm is introduced in one space dimension, which allows the recovery of solutions with concentrations, just by minimizing the modified action.

L2 approaches to some hyperbolic conservation laws

(This, roughly, covers contributions [6, 11, 20].)
The general form of multidimensional nonlinear conservation laws is:

$$\partial_t u + \sum_{i=1}^{d} \partial_i(F_i(u)) = 0,$$

where $u(t,x) \in V \subset R^m$ is a time dependent vector-valued field defined on a $d-$ dimensional domain and each $F_i : V \subset R^m \to R^m$ is a given nonlinear function. This includes systems of great importance in Mechanics and Physics, modeling gas dynamics or magnetohydrodynamics, for example. Many systems of physical origin have a variational origin and enjoy, from Noether's invariance theorem, an additional conservation law:

$$\partial_t(U(u)) + \sum_{i=1}^{d} \partial_i(G_i(u)) = 0, \tag{1}$$

where U and G_i are scalar functions (depending on F). When U is a strictly convex function, usually called "entropy function', the system automatically gets (locally) well-posed in suitable function spaces. However, solutions are expected to become discontinuous in finite time, even for smooth initial conditions (unless the system enjoys "linear degeneracy" or satisfies "null conditions"). There is no theory available to solve the initial value problem in the large, except in two extreme situations. First, for a single space variable ($d = 1$) and small initial conditions (in total variation), global existence and uniqueness of 'entropy solutions' have been established through the celebrated results of J. Glimm (existence) and A. Bressan and collaborators (well posedness). Next, for a *single* conservation laws ($m = 1$), global existence and uniqueness of "entropy solutions' have been established by Kruzhkov. In both cases, the L^1 space, as well as the companion space BV of bounded variation functions, plays a crucial role. One of the main difficulty of the field of non-linear hyperbolic conservation laws is the conflicting roles played by L^1 and L^2. Indeed, among all L^p spaces, the only convenient exponent for both Kruzhkov and Glimm-Bressan theories is $p = 1$, due to the treatment of shock waves, meanwhile, for generic multidimensional linear systems, such as the regular linear wave equation, well posedness requires $p = 2$ (Brenner 1966). However, we have found several cases for which, with the help of suitable changes of variables or coordinates, a hilbertian framework can be restored and used even for the treatment of singularities. The first examples are rather simple: one-dimensional Chaplygin gas, the Born-Infeld system in some simple situations, multidimensional scalar conservation laws [11, 20]. In all three cases, the method heavily relies on the trivial integrability of smooth solutions with the help, through an appropriate change of variables, of a simple linear PDE (of wave or advection type), well-posed in L^2. Then, global solutions are built beyond singularity formations just by adding a suitable non-linear but convex barrier potential to the linear operator, without loosing the well-posedness in L^2. This turns out to be equivalent to the "entropy condition". For example, any entropy solution $u(t,x)$ of a multidimensional scalar conservation law

$$\partial_t u + \nabla \cdot (G(u)) = 0,$$

valued in $[0,1]$, can be written

$$u(t,x) = \int_0^1 1\{Y(t,x,a) < 0\} da,$$

where Y solves the straightforward sub-differential inclusion (well-posed in L^2)

$$0 \in \partial_t Y + G'(a) \cdot \nabla Y \ + \partial \Phi[Y]$$

with initial consition $a - u(0,x)$, where $\Phi[Y]$ takes value 0 if $\partial_a Y \geq 0$ and $+\infty$ otherwise. This new formulation is very simple and fully compatible with the L^2 setting. Recently, using similar tools, we found [6] a possible multidimensional generalization of one-dimensional concave scalar conservation laws, which involves the Monge-Ampère equation and looks like a fully non-linear version of some popular models in chemotaxis theory or in astrophysics. For this model too, we get global well-posedness in L^2.

Hidden convexity in some relativistic equations

(This roughly covers contributions [14, 16, 17, 22].)
The Born-Infeld model is a beautiful nonlinear system of conservation laws for electromagnetism, which interpolates the Maxwell equations, for electromagnetic fields of low intensity, and foliations of the Minkowski space by classical strings, for fields of high intensity [16, 23]. (This is, by the way, related to the concept of "optimal transportation of currents", as explained in [23].) This system is known to be lorentzian, hyperbolic (well-posed), linearly degenerate and satisfies some null conditions. It admits an "entropy function", which, unfortunately, is convex only for fields of low intensity. We observed in [22] that this system can be enlarged (just by adding to it the conservation laws of the energy-momentum tensor provided by Noether's theorem, while ignoring their algebraic dependence on the electromagnetic field) as a very peculiar system of conservation laws, which

i) has the classical, galilean invariant (!), structure of MHD equations,
ii) admits a global convex "entropy functions" (first occurence of a "hidden convex structure")
iii) is weakly L^∞-star stable, at least for solutions depending one a single space variable.
As an output of this last property, we introduced the concept of "subrelativistic strings" as a natural weak L^∞-star completion of the usual concept of classical relativistic string [14]. This amounts to relaxing the algebraic relationship between fields, energy and momentum, and substituting for an algebraic manifold its convex hull (second occurence of a "hidden convex structure") In addition, let us mention that the apparently singular limit of the Born-Infeld system for very intense fields leading to classical strings just become regular, provided we use the enlarged system, written in non-conservative variables [17].

Optimal transport, rearrangement and convection

(This roughly covers contributions [9, 10].)
A popular statement of "optimal transport theory" is the fact that every L^2 map from a (nice) domain D of R^d to R^d admits a unique rearrangement with convex potential [48]. This result has been related to Fluid Mechanics through the study of minimizing geodesics on group of diffeomorphisms from the very beginning [51]. (See also [13, 29, 41].) It is only recently [10] that I found a direct connection between this monotone rearrangement theorem and the theory of Convection in natural sciences. [Convection is one of the most important mechanism in Nature, crucial for atmosphere, ocean, continental drift, volcanism, earthquakes...] Monotone rearrangement theory is about reorganizing a given function (or map) in some specific order (monotonicity, cycle monotonicity etc...). This is somewhat similar to convection, where fluid parcels are continuously reorganized in a stabler way (heavy fluid at bottom and light fluid at top). These analogies have been made more precise by analyzing the singular limit of Navier-Stokes equations with buoyancy and Coriolis forces, when a small source terms (heat, salinity) is acting on a long period of time. (This is partly a joint work with Mike Cullen, from the UK Met'office at Exeter.) Here, again, convexity plays a crucial role:
i) the limiting process (small source term versus long time interval) is well behaved as long as the pressure field stays uniformly strictly convex, in the limit equations,
ii) the existence (and the uniqueness, in some special cases) of global "entropy" solutions, with convex pressure field (in the large sense), can be proven for the limit equations.

Minimizing geodesics in inviscid fluid mechanics

This work is contained in [5].

There are very few problems in mathematical fluid mechanics for which global existence and uniqueness results are known without any restriction on the data. (For instance, concerning the Cauchy problem for the Navier-Stokes equation, we have global existence and uniqueness of solutions for L^2 initial conditions only in 2D, while the Leray global existence result in 3D lacks uniqueness; Concerning the Euler equations, global existence and uniqueness of solutions of the Cauchy problem have been proven by Yudovich in 2D and requires bounded vorticity; For gas dynamics, the Glimm-Bressan well-posedness theory is valid in 1D and for initial data of small total variation, etc...). One of them is the problem of minimizing geodesics ('shortest paths') on the group of 3D volume preserving diffeomorphisms with L^2 metric, which, following Arnold's geometric interpretation, is an alternative way of finding solutions to the Euler equations, with respect to the Cauchy problem. It is known (cf. [41] and recent improvements by Ambrosio and Figalli), that there is a unique possible pressure gradient for the minimizing curves whenever their end points are fixed. In addition, this pressure field has a limited but unconditional (internal) regularity: $p(t,x)$ is locally in $L^2_t(BV_x)$. This follows from the hidden convex structure of the problem, once written in terms of the pressure field. Our recent work [5] completes these results by showing:

1) the uniqueness property is false in the case of finite dimensional configuration spaces such as $O(3)$ for the motion of rigid bodies, and, therefore, can be viewed as an infinite dimensional phenomenon (related to the possibility of relaxing the corresponding minimization problem by convex optimization);

2) the unconditional partial regularity is necessarily limited: we found explicit examples of pressure $p(t,x)$ fields with infinite curvature in x.